MATH 582 HOMEWORK 4
WEEK 8
Winter, 2009
Due March 13

Exercise 1. *A set z is a transitive set if and only if $\bigcup z \subseteq z$.*

Exercise 2. *Give an example of sets x and y satisfying the following: y is a transitive set and $x \in y$ but x is not a transitive set.*

Exercise 3. *If X is a nonempty set of ordinals, then $\bigcap X$ is an ordinal, and the least element of X.*

Exercise 4. *Let $X \subset \mathrm{ON}$.*
(a) $\bigcup X$ is an ordinal.
(b) $\bigcup X$ is an upper bound for X: for all $\alpha \in X$, $\alpha \leq \bigcup X$.
(c) $\bigcup X$ is the least upper bound for X: for all ordinal γ, if γ is an upper bound for X, then $\bigcup X \leq \gamma$.
(d) Suppose X is nonempty and has no greatest ordinal. Show $\bigcup X$ is a limit ordinal.
(e) Show: There is an ordinal α such that $\alpha > \beta$ for all $\beta \in X$. (So, $\alpha \notin X$, and this provides another proof that there is no set X containing all ordinals.)

Exercise 5. *If x is a finite ordinal then so is every $y \in x$, and $S(x)$ is also a finite ordinal. Furthermore, every natural number is a finite ordinal.*