MATH 582 HOMEWORK 2
WEEK 3

Winter, 2009

Due February 6

1. The language of sets consists of a single binary relation $\{\epsilon\}$. An *interpretation* (or a *model*) of the language of sets consists of a pair (D, E), where D is the *domain of discourse* and E is a binary relation on D which is the interpretation of the membership relation ϵ. Once we provide an interpretation of the language of sets, all sentences in the language become true or false.

The simplest way of describing the relation E (for finite models) is as a *directed graph* (or a *digraph*.) The domain of discourse D is the nodes of the graph, and the edges of the graph (represented by arrows) determine the membership relation E. If x and y are nodes of the graph then $x \in y$ is true if there is an arrow from x to y; and, $x \notin y$ if there is no arrow from x to y. For example, in the model **1e** below we have only $a \in b$ and $a \in c$ (so, $a \notin a$, $b \notin b$, $b \notin c$, $b \notin a$, $c \notin a$, $c \notin b$, $c \notin c$.) In this model the Axiom of Foundation is true:

- It holds for a because a has no elements (so the antecedent of the Axiom fails.)
- It holds for b because only $a \in b$, but a and b share no members (so the consequent of the Axiom is true.)
- It holds for c because only $a \in c$, but a and c share no members (so the consequent of the Axiom is true.)

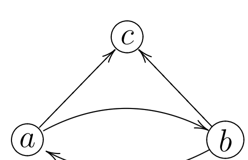
On the other hand, the Axiom of Extensionality is false. I leave it to you to show why.

For each of the following interpretations of set theory state which of axioms 1,2,4,5 are true. Explain your answer.

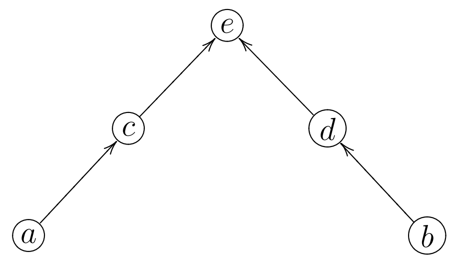
1a. $D = \{a\}$, E is 

1b. $D = \{a\}$, E is 

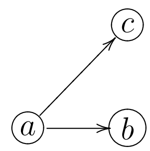
1c. $D = \{a, b, c\}$, E is



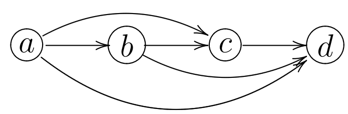
1d. $D = \{a, b, c, d, e\}$, E is



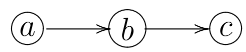
1e. $D = \{a, b, c\}$, E is



1f. $D = \{a, b, c, d\}$, E is



1g. $D = \{a, b, c\}$, E is



2. Which of the models in **1** satisfies Axiom of Extensionality and the statement that there is no empty set ($\neg\exists x \forall y (y \notin x)$.)

3. Show that model **1a** satisfies the Axiom of Comprehension.

4. Which of the models in **1** satisfies the Axioms of Extensionality and Comprehension but does not have pairwise unions – that is, the model will contain elements z, u but will contain no w satisfying $\forall x (x \in w \leftrightarrow x \in z \vee x \in u)$.