

MATH 582 HOMEWORK 1

WEEK 2

Winter, 2009

Due January 23

**Exercise 1** Show that the following identity need not hold when  $f : A \rightarrow B$  is not injective.

$$f[X \cap Y] = f[X] \cap f[Y] \quad X, Y \subseteq A.$$

**Exercise 2** Prove the following identities. For every  $f : A \rightarrow B$  and all sequences of sets  $Y_n \subseteq B$  and  $X_n \subseteq A$ ,

$$(a) \quad f^{-1}\left[\bigcup_{n=0}^{\infty} Y_n\right] = \bigcup_{n=0}^{\infty} f^{-1}[Y_n],$$

$$(b) \quad f^{-1}\left[\bigcap_{n=0}^{\infty} Y_n\right] = \bigcap_{n=0}^{\infty} f^{-1}[Y_n],$$

$$(c) \quad f\left[\bigcup_{n=0}^{\infty} X_n\right] = \bigcup_{n=0}^{\infty} f[X_n],$$

**Exercise 3** Prove. For every injection  $f : A \hookrightarrow B$  and every sequence of sets  $X_n \subseteq A$ ,

$$f\left[\bigcap_{n=0}^{\infty} X_n\right] = \bigcap_{n=0}^{\infty} f[X_n]$$

**Exercise 4** Let  $A_n$  and  $B_n$  be sequences of sets. Prove the following identities.

$$(a) \quad \forall n [A_n \subseteq C] \rightarrow \bigcup_{n=0}^{\infty} A_n \subseteq C,$$

$$(b) \quad \bigcup_{n=0}^{\infty} (A_n \cap C) = \left(\bigcup_{n=0}^{\infty} A_n\right) \cap C,$$

$$(c) \quad \bigcup_{n=0}^{\infty} (A_n \cup B_n) = \bigcup_{n=0}^{\infty} A_n \cup \bigcup_{n=0}^{\infty} B_n.$$