## Midterm 2 A

(a) If $x$ does not occur free in $\gamma$, $\models \big((\forall x)(P(x) \leftrightarrow \gamma\big) \to \big((\forall x)P(x) \lor (\forall x)\neg P(x)\big)$.

(b) If $x$ does not occur free in $\gamma$, $\vdash \big((\forall x)(P(x) \leftrightarrow \gamma\big) \to \big((\forall x)P(x) \lor (\forall x)\neg P(x)\big)$. (That is, provide a tableaux proof.)

## Midterm 2 B

Let $\Sigma$ be a set of first-order *quantifier-free* sentences whose constants all come from the set $A = \{a_0, a_1, \ldots\}$.

> Prove the following: If there is a Boolean assignment in which all the sentences of $\Sigma$ are true, then there is a first-order assignment in which all the sentences of $\Sigma$ are true.

*Hint*. Start with a Boolean assignment in which all the sentences of $\Sigma$ are true, and use this to produce a first-order structure. Then prove that every sentence of $\Sigma$ is true in this structure by structural induction.

## Natural Deduction

(a) $\vdash_{\text{nd}} (\forall x)\big((P(x) \to Q(x)) \to (\neg Q(x) \to \neg P(x))\big)$

(b) $\vdash_{\text{nd}} (\forall x)(P(x) \lor Q(x)) \to (\exists x)P(x) \lor (\forall x)Q(x)$.

(c) $\vdash_{\text{nd}} (\exists x)(P(x) \to (\forall x)P(x))$.

(d) $(\forall x)(P(x) \to Q(x)), (\forall x)(Q(x) \to R(x)) \vdash_{\text{nd}} (\forall x)(P(x) \to R(x))$

(e) $(\exists x)(P(x) \lor Q(x)), (\forall x)\neg P(x) \vdash_{\text{nd}} (\exists x)Q(x)$

(f) $(\forall x)(I(x,x) \to (\exists x)\neg I(x,y)), \vdash_{\text{nd}} \neg(\exists x)(I(x,x) \land (\forall y)I(x,y)$

## Natural Deduction

Formulate an introduction and elimination rule for the biconditional in natural deduction and use it to prove the following.

(a) $\vdash_{\text{nd}} (\forall x)P(x) \leftrightarrow \neg(\exists x)\neg P(x)$

(b) $\vdash_{\text{nd}} (\exists x)P(x) \leftrightarrow \neg(\forall x)\neg P(x)$