Math481 Homework 1 due Monday, November 14

The problems on this assignment are required of everyone. I will put together a second assignment with advanced problems that will be due after the break.

Symbolizing Arguments

Symbolize the following argument into first-order logic. Then determine whether the conclusion is a logical consequence of the premises by using the definition of logical consequence.

Define a *lover* as anyone who loves at least one person. Suppose we are given the following facts

- Everyone loves a lover.
- Someone is a lover.

Does it follow that everyone loves each other?

Valid Inferences

Show the following inferences are valid.

(a) If $\Sigma, \phi(c) \models \psi$, then $\Sigma, (\exists x)\phi(x) \models \psi$, where c is a constant which does not occur in any of the sentences in Σ nor in $\phi(x), \psi$.

(b) If $\Sigma \models \phi(c)$ then $\Sigma \models (\forall x)\phi(x)$, where c is a constant which does not occur in any of the sentences in Σ nor in $\phi(x)$.

0.1 Semantic Tableaux

Use semantic tableaux to test whether the following are valid. If not, then provide a structure in which the sentence is false.

1. $(\exists x)(\forall y)R(x, y) \rightarrow (\forall y)(\exists x)R(x, y)$.

2. $(\exists x)(P(x) \rightarrow (\forall x)P(x))$.

3. $(\forall x)(P(x) \rightarrow Q) \rightarrow ((\forall x)P(x) \rightarrow Q)$, where x does not occur free in Q.

4. $(\forall x)(\exists y)(\forall z)(\exists w)\big(R(x,y) \vee \neg R(w,z)\big)$.

5. $(\exists x)(\forall y)(\forall z)(\forall w)\big((P(y) \vee Q(z) \vee R(w)) \rightarrow (P(x) \vee Q(x) \vee R(x))\big)$

6. Use Semantic tableau to test the validity of the argument in the first section.