

Exercise 1

Which of the following expressions are official (that is, unabbreviated). Prove your answer by either giving a step by step derivation of the expression or proving, by induction on propositions, that there is some property enjoyed by all propositions but not this expression.

(a) $((\neg(P \vee Q)) \wedge R)$

(b) $(P \wedge Q) \vee R$

(c) $((Q \vee R) \wedge P) \leftrightarrow Q$

Exercise 2

Define using structural recursion a function d on propositions, so that d returns the number of logical connectives occurring in the proposition. (If α is a propositional atom, then $d(\alpha) = 0$.)

Exercise 3

Which of the following are tautologies. If they are not a tautology, provide an assignment which establishes this. The truth table for \uparrow (read as “not both” or NAND) is as follows:

P	Q	$P \uparrow Q$
T	T	F
T	F	T
F	T	T
F	F	T

(a) $((P \rightarrow Q) \wedge (Q \leftrightarrow \top)) \rightarrow (P \leftrightarrow \top)$

(b) $((\neg(P \wedge Q)) \wedge (P \leftrightarrow \perp)) \leftrightarrow (\neg(Q \leftrightarrow \perp))$

(c) $((P \rightarrow Q) \uparrow (P \rightarrow Q)) \leftrightarrow (P \wedge \neg Q)$

(d) $((P \uparrow Q) \uparrow (P \uparrow Q)) \leftrightarrow (P \wedge Q)$

(e) Find a proposition using the propositional letters P and Q and any of the connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ with the same truth table as $P \uparrow Q$.

Exercise 4

The island of knights and knaves has the following laws: (i) everyone is a knight or knave, (ii) knights only speak truly, (iii) knaves only speak falsely.

Use the following translation key:

k_1 : A_1 is a knight.

k_2 : A_2 is a knight.

G : There is gold on the island.

and translate the following conversations as a pair of statements. In each case state which of the propositions k_1 , k_2 and G must be true, which must be false, or either truth value is possible.

(a).

A_1 If A_2 is a knight, then there is gold on the island.

A_2 If A_1 is a knave, then there is gold on the island.

(b).

A_1 If I am a knight and A_2 is a knave, then there is gold on the island.

A_2 That is not true!

(c).

A_1 If we are of the same type, then there is gold on the island.

A_2 We are not of the same type.