MATH 425
Midterm 2
Winter, 2009

Name:

- You have **50 minutes** to complete your work.
- Show all work and make it clear what your answers are.
- You are permitted two 3x5 notecard. Otherwise, books, notes, calculators and computers are not permitted on this exam.

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problem	points	score
1	20	
2	20	
3	15	
4	10	
5	10	
6	10	
Total	85	

1. (20 points)
For each of the random variables X below, determine the type of distribution which best models X. Give the values of the parameters to the distribution you have chosen. State your assumptions you think relevant for your choice. Space is given for your answers on the next page.

a. In a class of 80 students, the professor calls on 2 students chosen at random for a recitation in each class period. There are 40 class periods in a term. Let X be the random variable which counts the number of times student s is called in a term.

b. A small college has found that on average only 1 in every 3 students accepted will actually attend. They accept 450 students for next year's class. Let X count the number of students who actually attend.

c. During the peak time of the Capricornids meteor shower, the average time between meteors is 4 minutes. Let X count the time (in hours) you wait until you observe 25 meteors.

d. Smith is playing craps and tossed a 5 on his first throw. He must continue tossing until either a 5 appears (and he wins) or a 7 appears (and he loses). Let X count the number of throws (not counting Smith's first throw) until a winner is determined.

1. Work space.

a.

b.

c.

d.

2. (20 points)

a. Let X be a uniformly distributed random variable over $[0, 2]$. Find the probability density function for $Y = e^X$.

b. Let X be a random variable with $E[X] = 2$ and $Var(X) = 3$, and $Y = 3X + 4$. Find the expectation and variance of Y.

c. Let X be a normally distributed random variable with $\mu = 4$ and $\mathbf{P}\{X > 0\} = 0.7881$. Find the variance σ^2 for X.

d. Let X be a binomial random variable with expected value 2.0 and variance 1.2. Find the parameters n and p for X.

3. (15 points)

For some constant c, the random variable X has the probability density function

$$f_X(x) = \begin{cases} c(1 - x^2) & \text{if } -1 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find c.

(b) Compute $E[X]$.

(c) Compute $Var(X)$.

4. (10 points)
Bippy's Bearing Works manufactures bearing shafts whose diameters are normally distributed with mean 1 and standard deviation 0.002 cm. Their specifications require the diameters to be in the range 1.000 ± 0.003 cm.

(a) What percentage of shafts are rejected?
The probability must be given to four decimal places.

(b) What is the maximum standard deviation that will permit no more than 1% of their shafts to be rejected?

5. (10 points)
The number of hours of use of a lightbulb is exponentially distributed, with an expected lifetime of 1000 hours.

(a) Compute the probability that the light bulb is still working after 1000 hours.

(b) Compute the probability that a light bulb has at least another 500 hours of use given that it has already been in use 500 hours.

6. (10 points)
You are submitting a sealed bid to paint a house. If you have the lowest bid, then you will win the contract. You intend to pay Acme Painting 5 thousand dollars to do the work. If you believe the lowest competing bid (in thousands of dollars) of the other participating contractors will be uniformly distributed in the interval $(4, 9)$, how much should you bid to maximize your profit?