

MATH 425
Midterm 1
Winter, 2009

Name:

-
- You have **50 minutes** to complete your work.
 - Show all work and make it clear what your answers are.
 - You are permitted one 3x5 notecard. Otherwise, books, notes, calculators and computers are not permitted on this exam.
-

problem	points	score
1	15	
2	15	
3	10	
4	10	
5	20	
Total	70	

1. (5 points each)

a. How many arrangements of the letters REMEMBER ?

b. How many 5 digit numbers can be formed from the digits $\{1, 2, \dots, 9\}$ if no digit can appear more than twice?

c. How many 5 digit numbers can be formed from the digits $\{1, 2, \dots, 9\}$ if no two consecutive digits can be the same?

2. (15 points) Consider three events A, B, C with

$$\mathbf{P}(A) = 0.4 \quad \mathbf{P}(B) = 0.5 \quad \mathbf{P}(C) = 0.7.$$

- a. (2 pts) What is $\mathbf{P}(A|B)$ if A and B are independent?

- b. (2 pts) What is $\mathbf{P}(A|B)$ if A and B are mutually exclusive?

- c. (2 pts) What is $\mathbf{P}(A|B)$ if $\mathbf{P}(A \cap B) = 0.2$?

- d. (2 pts) What is the smallest possible value of $\mathbf{P}(B \cap C)$?

- e. (2 pts) What is the largest possible value of $\mathbf{P}(B \cap C)$?

- f. (5 pts) What is $\mathbf{P}(A \cup B \cup C)$ if each of A, B and C are independent?

3. (5 points each)

a. A person chooses a letter at random from RESERVE, and independently one at random from VERTIGO. What is the probability that the same letter is chosen.

b. A forest preserve contains 20 elk, of which 5 are captured, tagged and released. A certain time later 4 of the elk are captured. What is the probability that 2 of the 4 have been tagged? (Assume that each elk is equally likely to be captured at any time.)

4. (10 points) Urn A contains 3 white balls and 2 black balls. Urn B contains 1 white ball and 4 black balls. A ball is drawn at random from urn A and placed into urn B . Urn B is thoroughly mixed and a ball is drawn.

- (a) Suppose a white ball is drawn from urn B . What is the probability that the ball transferred from urn A is white?
- (b) Suppose a black ball is drawn from urn B . Now, what is the probability that the ball transferred from urn A is white?

5. (20 points) The color of a person's eyes is determined by a single pair of genes. If they are both blue-eyed genes, then the person will have blue eyes; if they are both brown-eyed genes, then the person will have brown eyes; and if one of them is a blue-eyed gene and the other a brown-eyed gene, then the person will have brown eyes. A newborn child independently receives one eye gene from each parent and the gene it receives from a parent is equally likely to be either of the two eye genes of that parent. Suppose that Smith's sister and mother have blue eyes, but Smith's father has brown eyes.

(a) What is the probability that Smith has blue eyes?

Suppose Smith's wife has blue eyes.

(b) What is the probability that their first child will have blue eyes?

(c) If their first child has blue eyes, what is the probability that their next child will also have blue eyes?

(d) If their first child has brown eyes, what is the probability that their next child will also have brown eyes?