MATH 425
HOMEWORK 1
Winter, 2009

1 Chapter 1 Problems

2. There are $6^4 = 1296$ possible outcomes a a die thrown four times.

3. There are $20! \approx 2.433 \times 10^{18}$ possible ways of assigning twenty workers twenty different jobs.

5. There are $8 \cdot 2 \cdot 9 = 144$ possible areas codes.

8.

(a) There are $5! = 120$ arrangements of the letters in 'fluke'.

(b) There are $\binom{7}{2,2} = \frac{7!}{2! \cdot 2!} = 1260$ arrangements of the letters in 'propose'.

(c) There are $\binom{11}{4,4,2,1} = \frac{11!}{4! \cdot 4! \cdot 2!} = 34,650$ arrangements of the letters in 'mississippi'.

(d) There are $\binom{7}{2,2} = \frac{7!}{2! \cdot 2!} = 1260$ arrangements of the letters in 'arrange'.

10.

(a) There are $8! = 40,320$ arrangements of eight people in a row.

(b) There are $2 \cdot 7 \cdot 6! = 10,080$ ways of seating eight people in a row if two (A and B) must sit together.

(c) There are $2 \cdot 4! \cdot 4! = 1152$ ways of arranging the 4 men and 4 women so that men and women alternate.

(d) There are 4 ways to seat the five men together, so there are $4 \cdot 5! \cdot 3! = 2880$ of seating five men together in the eight people.

(e) There are 4! ways of arranging four couples. Within each arrangement of couples there are 2^4 arrangements of the individuals. So, there are $4! \cdot 2^4 = 384$ arrangements of the 4 couples and eight people.

15. There are $\binom{10}{5} \cdot \binom{12}{5} \cdot 5!$ ways of pairing off 5 women from 10 and 5 men from 12.

19.

(a) There are $\binom{8}{3} \cdot \binom{4}{3} + \binom{2}{1} \cdot \binom{8}{3} \cdot \binom{4}{2} = 896$ ways to choose 3 women from 8 and 3 men from 6, when 2 men refuse to work together.

(b) There are $\binom{6}{3} \cdot \binom{6}{3} + \binom{2}{1} \cdot \binom{6}{2} \cdot \binom{6}{3} = 1000$ ways to choose 3 women from 8 and 3 men from 6, when 2 women refuse to work together.

(c) There are $\binom{7}{3}\binom{5}{3} + \binom{7}{2}\binom{5}{3} + \binom{7}{3}\binom{5}{2} = 910$ ways to choose 3 women from 8 and 3 men from 6, when 1 woman and 1 man refuse to work together.

24. Use the binomial theorem:

$$\begin{aligned}(3x^2+y)^5 &= \binom{5}{0}3^5x^{10} + \binom{5}{1}3^4x^8y + \binom{5}{2}3^3x^6y^2 + \binom{5}{3}3^2x^4y^3 + \binom{5}{4}3x^2y^4 + \binom{5}{5}y^5 \\ &= 243x^{10} + 405x^8y + 270x^6y^2 + 90x^4y^3 + 15x^2y^4 + y^5\end{aligned}$$

30. There are two ways to approach the problem (and of course, they lead to the same solution):

1. Count how many ways there are to seat just the British and French delegates together and the Russian and American apart, and multiply this value by 6!. I don't know an easy way to do this, except the painful one: count them out.

2. Count the ways to seat the British and French delegates together and everyone else wherever: $9 \cdot 2! \cdot 8!$. Now, subtract out the number of ways that the American and Russian delegates are seated together. I can count this quickly, so I will explain how.

Seating the English-French delegate at positions (1,2) or (9,10) allow 7 possible arrangements (each) for the American-Russian delegate to sit together (in some order). Otherwise, the English-French delegate are at some position $(n, n+1)$ where $2 \leq n \leq 8$. In each case the number of positions available for sitting the American-Russian delegates together (in some order) is:

$$\big[(n-2) + (10 - (n+2))\big] = 6$$

Here is the reason:

- There are $n-2$ possible positions before the English-French pair (they could start at any position 1 to $n-2$).
- There are $10-(n+2)$ possible positions after the English-French pair (they could start at any position from $n+1$ to 9).

So, there are a total of 56 possible placements of the American-Russian ambassadors that seat them together, when the English-French ambassadors are sitting together. Once these placements are made, there are $2! \cdot 2! \cdot 6!$ arrangements of the delegates.

The total number of arrangements then is

$$9 \cdot 2! \cdot 8! - 56 \cdot 2! \cdot 2! \cdot 6! = 7 \cdot 2! \cdot 8! = 564,480.$$