

MATH 417
Supplemental Midterm 2
Winter, 2008

Name:

Class time:

-
- **Due** Monday, March 17 at the beginning of class.
 - You may use your text, notes or calculator. You **must** work alone.
 - Show your work, document your reasons for the solution. No credit will be given if we cannot verify the process that led you to your solution.
 - Check your work carefully; you are responsible for your computations. No partial credit will be given for mistakes in computation.
-

problem	points	score
1	5	
2	15	
3	10	
Total	30	

1. Find a basis \mathcal{B} for \mathbb{R}^4 such that the following matrix represents the identity transformation between the standard basis and \mathcal{B} :

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 2 & 2 \\ -1 & 1 & -1 & 2 \end{bmatrix}$$

2. Let \mathcal{B} be a basis for \mathcal{R}^3 consisting of the following vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation given by

$$T(\vec{v}_1) = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \quad T(\vec{v}_2) = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \quad T(\vec{v}_3) = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$$

- (a) Find the matrix for T relative to the standard basis.
- (b) Find the matrix for T relative to \mathcal{B} .
- (c) Find the matrix for T relative to the following basis, given in coordinates for \mathcal{B} :

$$[\vec{u}_1]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad [\vec{u}_2]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad [\vec{u}_3]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

3. Find orthonormal bases for the following subspace and for its orthogonal complement:

$$V = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} : x - y - z + w = 0 \text{ and } x + z = 0 \right\}$$