MATH 417
Midterm 1 – Study Guide
Winter, 2008

Study Guide: Content

The material will be drawn from Chapters 1, 2, 3.1, and 3.2 (although you will be expected to be able compute a basis for the kernel and image of a matrix.) You will want to review problems from the homework, in addition to the problems below (which are intended for additional practice.)

Computational Problems

Gauss-Jordan Elimination. Exercises from 1.2: 1-17.

Computing Inverses. Exercises from 2.3: 1-20.

Matrix Product. Exercises from 2.4: 1-14.

Linear Independence. Exercises from 3.2: 10-26.

Basis for Image and Kernel. Exercises from 3.3: 1-25.

True/False Problems

The problems can be found at the end of each chapter. Solutions to all odd numbered true/false problems can be found on the author's website:

http://www.colby.edu/personal/o/obretsch/ssm/

Chapter 1. 17, 21, 29, 30, 36, 37, 39, 41, 43

Chapter 2. 8, 15, 21, 29, 36, 43, 44, 45, 52

Conceptual

The following problems test your understanding of fundamental concepts introduced in chapter 3. Solutions can be found in the Chapter and in lecture notes. You need to be able to prove these facts.

1. Let $\vec{v}_1, \ldots, \vec{v}_k$ be vectors in $\mathbb{R}^n$. Then $\text{span}\big(\vec{v}_1, \ldots, \vec{v}_k\big)$ is always a subspace.

2. Let A be an $n \times m$ matrix. Then the image of A is a subspace of $\mathbb{R}^n$ and the kernel of A is a subspace of $\mathbb{R}^m$.

3. If $\vec{v}_1, \ldots, \vec{v}_k$ are linearly independent vectors of $\mathbb{R}^n$ and $\vec{v}$ is a vector of $\mathbb{R}^n$ with
$$\vec{v} \notin \text{span}\big(\vec{v}_1, \ldots, \vec{v}_k\big);$$
then $\vec{v}_1, \ldots, \vec{v}_k, \vec{v}$ are linearly independent

4. If $\vec{v}$ is a nonzero vector and $\vec{v} \in \text{span}\big(\vec{v}_1, \ldots, \vec{v}_k\big)$ then $\vec{v}_1, \ldots, \vec{v}_k, \vec{v}$ linearly dependent.

5. Every subspace V of $\mathbb{R}^n$ has a basis with no more than n vectors.

6. If $\vec{v}$ is a linear combination of $\vec{v}_1, \ldots, \vec{v}_k$ then
$$\text{span}\big(\vec{v}_1, \ldots, \vec{v}_k, \vec{v}\big) = \text{span}\big(\vec{v}_1, \ldots, \vec{v}_k\big)$$

7. If $\vec{v}_1, \ldots, \vec{v}_k$ is a basis for a subspace V of $\mathbb{R}^n$ then there is exactly one way to write any vector $\vec{y}$ in V as a linear combination of $\vec{v}_1, \ldots, \vec{v}_k$.