MATH 417
HOMEWORK 1
Winter, 2008

Section1.1

26. Applying Gauss-Jordan elimination to the augmented matrix we reach the following point

$$\begin{bmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & (k^2-5) & k \end{bmatrix} \xrightarrow{Gauss-Jordan} \begin{bmatrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & (k^2-4) & k-2 \end{bmatrix}$$

We can divide by $k^2 - 4$, provided this is non-zero. Suppose $k \neq 2, -2$. Then we can continue Gauss-Jordan elimination to the following reduced row-echelon form:

$$\begin{bmatrix} 1 & 0 & 0 & \frac{k+5}{k+2} \\ 0 & 0 & 1 & \frac{k}{k+2} \\ 0 & 0 & (k^2-5) & \frac{1}{k+2} \end{bmatrix}$$

This system of equations has a unique solution, $(\frac{k+5}{k+2}, \frac{k}{k+2}, \frac{1}{k+2})$.

If $k = -2$ the reduced row echelon form is

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which is inconsistent.

If $k = 2$ the reduced row echelon form is

$$\begin{bmatrix} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

which has infinitely many solutions: $(3z + 1, 1 - 2z, z)$ for any real number z.

32. The system of equations is as follows:

$$\begin{aligned} a + b + c &= 1 \\ 1 + 2b + 4c &= 0 \\ a + \frac{3}{2}b + \frac{7}{3}c &= -1 \end{aligned}$$

The polynomial is $f(t) = 20 - 28t + 9t^2$.

Section 1.2

4. The augmented matrix after Gauss-Jordan elimination:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 5 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{Gauss-Jordan} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

So, $(2, -1)$ is the unique solution.

8. The augmented matrix after Gauss-Jordan elimination:

$$\begin{bmatrix} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 4 & 8 & 0 \end{bmatrix} \xrightarrow{Gauss-Jordan} \begin{bmatrix} 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{bmatrix}$$

So, there are infinitely many solutions: $(x_1, x_5, x_3, -2x_5, x_5)$ for all real numbers x_1, x_3, x_5.

10. The augmented matrix after Gauss-Jordan elimination:

$$\begin{bmatrix} 4 & 3 & 2 & -1 & 4 \\ 5 & 4 & 3 & -1 & 4 \\ -2 & -2 & -1 & 2 & 3 \\ 11 & 6 & 4 & 1 & 11 \end{bmatrix} \xrightarrow{Gauss-Jordan} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -3 & 2 \\ 0 & 0 & 1 & 2 & -33 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

There are infinitely many solutions: $(1 - x_4, 2 + 3x_4, -3 - 2x_4, x_4)$ for all real numbers x_4.

42. We will use the following variables (starting at the top and going clockwise.)

1. x : traffic volume on Mt. Auburn between JFK and Dunster,
2. y : traffic volume on Dunster between Mt. Auburn and Winthrop
3. z : traffic volume on Winthrop between Dunster and JFK

4. w : traffic volume on JFK between Winthrop and Mt. Auburn

Each intersection has as many cars going in as coming out. This produces the following equations (starting at the intersection of JFK and Mt. Auburn and going clockwise):

$$\begin{aligned} w + 300 &= x + 400 \\ x + y + 100 &= 250 \\ 270 &= y + z \\ z + 300 &= w + 320 \end{aligned}$$

The augmented matrix after Gauss-Jordan elimination:

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 100 \\ 1 & 1 & 0 & 0 & 150 \\ 0 & 1 & 1 & 0 & 270 \\ 0 & 0 & 1 & -1 & 20 \end{bmatrix} \xrightarrow{Gauss-Jordan} \begin{bmatrix} 1 & 0 & 0 & -1 & -100 \\ 0 & 1 & 0 & 1 & 250 \\ 0 & 0 & 1 & -1 & 20 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So, we have the following constraints given by the problem:

$$\begin{aligned} x, y, z, w &\geq 0 \\ x &= w - 100 \\ y &= 250 - w \\ z &= w + 20 \end{aligned}$$

We must have $100 \leq w \leq 250$. The rest of x, y, z can be determined from the value of w.

46. Applying Gauss-Jordan elimination to the augmented matrix we reach the following point

$$\begin{bmatrix} 0 & 1 & 2k & 0 \\ 1 & 2 & 6 & 2 \\ k & 0 & 2 & 1 \end{bmatrix} \xrightarrow{Gauss-Jordan} \begin{bmatrix} 1 & 0 & 6 - 4k & 2 \\ 0 & 1 & 2k & 0 \\ 0 & 0 & (2k-2)(2k-1) & -(2k-1) \end{bmatrix}$$

We can divide by $(2k-2)(2k-1)$, provided this is non-zero. Suppose $k \neq 1, \frac{1}{2}$. Then we can continue Gauss-Jordan elimination to the following reduced row-echelon form:

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{-1+k} \\ 0 & 1 & 0 & \frac{k}{-1+k} \\ 0 & 0 & 1 & 1 - \frac{1}{2(-1+k)} \end{bmatrix}$$

This system of equations has a unique solution, $(\frac{1}{-1+k}, \frac{k}{-1+k}, 1 - \frac{1}{2(-1+k)})$.

If $k = 1$ the reduced row echelon form is

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1) \end{bmatrix}$$

which is inconsistent.

If $k = \frac{1}{2}$ the reduced row echelon form is

$$\begin{bmatrix} 1 & 0 & 4 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

which has infinitely many solutions: $(2 - 4z, -z, z)$ for any real number z.